Ballistic annihilation in a one-dimensional fluid

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A kinetic equation for the two-particle conditional distribution of nearest neighbors is derived as a rigorous consequence of the dynamics of ballistic annihilation. The equation describes completely the evolution of the system when at the initial moment higher order conditional distributions factorize into products of two-particle ones. The derived equation provides a rigorous analytic method to study the process of ballistic annihilation. To illustrate the method the two-velocity case is solved explicitly. It is shown that the annihilation dynamics in one dimension creates strong correlations between the velocities of colliding particles, which rules out the Boltzmann approximation.

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I. INTRODUCTION

We consider here a one-dimensional fluid composed of point particles. The motion between collisions is free. When two particles collide they instantaneously annihilate each other and disappear from the system. This kind of dynamics, called *ballistic annihilation*, has been the object of recent studies [1,2], motivated mainly by its potential relevance to the kinetics of chemical reactions. However, no rigorous analytic approach has been elaborated up to now. In order to determine the law of decay of the initial density, and the evolution of the velocity distribution of the reacting fluid, approximate schemes have been tried based on scaling arguments and on the kinetic equation of the Boltzmann type. An exception is the paper by Elskens and Frisch [3]. Supposing that the particles can move with two possible velocities $+c$ or $-c$ only, the authors developed a combinatorial analysis and found a number of rigorous results. In particular, in the case of a symmetric initial velocity distribution the density of the fluid was shown to decay as $t^{-1/2}$. However, the combinatorial approach could not be generalized to other initial states, and was not further developed.

It is the aim of the present paper to provide an analytic approach which permits one in principle to study rigorously the ballistic annihilation in a one-dimensional fluid for an arbitrary uncorrelated initial velocity distribution. The fundamental role in our analysis will be played by the distribution of nearest neighbors (the scaling analysis of the spatial interparticle distribution function in diffusion-limited reactions has been performed in [4]).

Suppose that at time t there is a particle at point x_1 in the fluid, moving with velocity v_1. We denote by

$$\mu(x_2, v_2|x_1, v_1; t) \tag{1}$$

the conditional probability density for finding its right nearest neighbor at distance $x_{21} = x_2 - x_1 > 0$, with velocity v_2. The density (1) satisfies the normalization condition

$$\int d2\ \mu(2|1;t) = 1, \tag{2}$$

where a convenient shorthand notation

$$j \equiv (x_j, v_j), \quad dj \equiv dx_j dv_j, \quad j = 1, 2, \ldots$$

has been used.

When the state of the fluid is translationally invariant $\mu(2|1;t)$ depends in the position space on the distance x_{21} only

$$\mu(x_2, v_2|x_1, v_1; t) = \mu(x_{21}, v_2|0, v_1; t).$$

A particular role will be played by the value of density μ at contact

$$\mu(0+, v_2|0, v_1; t) = \lim_{0<x\to 0} \mu(x, v_2|0, v_1; t). \tag{3}$$

The notation 0+ stresses the fact that the distance between the particles approaches zero through positive values. The special role of the contact value $\mu(0^+, v_2|0, v_1; t)$ comes from the fact that it determines the density of precollisional configurations.

In Sec. II we discuss survival probabilities of the initial free trajectories and express them in terms of function (3). The most important development is presented in Sec. III. We demonstrate therein that a closed nonlinear evolution equation for the distribution $\mu(2|1;t)$ follows rigorously from the assumed annihilation dynamics. This remarkable fact together with the derivation of the form of the evolution equation is the main result of the present work. The evolution equation is solved in Sec. IV in the two-velocity case. This permits one in particular to recover the results of the combinatorial analysis [3] of Elskens and Frisch. However, the derived equation is general, valid for arbitrary initial velocity distributions, both discrete and continuous. It opens thus a way to study the subtle dependence of the ballistic-annihilation process on the initial condition of the fluid. The paper ends with concluding remarks (Sec. V).

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II. SURVIVAL OF FREE TRAJECTORIES

The important characteristic of the annihilation dynamics is that only those particles which suffered no collisions are present in the system. They are thus found on their free trajectories. In this sense the terms *survival of a particle* and *survival of a free trajectory* are equivalent. Let us begin by considering a simple case where at the initial moment the particles are uniformly distributed in space according to the Poisson law, with no correlations between their velocities. The initial state of the fluid is thus translationally invariant, and each particle has at $t = 0$ the same probability density $\phi(v; t = 0)$ to start the motion with velocity v.

We denote by $S(v; t)$ the probability that a free trajectory corresponding to velocity v remains unperturbed during the time interval $[0, t]$. This event can occur only if the particle following the trajectory suffered no collision either from the left or from the right. The assumed absence of correlations between the velocities in the initial state implies the product structure

$$S(v;t) = S^L(v;t)S^R(v;t), \tag{4}$$

where $S^L(v; t)$ and $S^R(v; t)$ are the probabilities for the absence of collisions with the left and right neighbors, respectively. The translational invariance makes them independent of the position variable.

Let us study the evolution of the probability $S^R(v; t)$. Its value changes in the course of time due to collisions with particles arriving from the right hand side. Their distribution around particle $1 = (x_1, v_1)$ at time t is described by the conditional probability density $\mu(2|1; t)$. Introducing the Heaviside unit step function

$$\theta(x) = \begin{cases} 1, & \text{if } x > 0 \\ 0, & \text{if } x < 0 \end{cases} \tag{5}$$

we thus find that the expression

$$\theta(x_{21})\theta(x_{12} + v_{12}dt)\mu(2|1;t) \tag{6}$$

represents for $dt \to 0+$ the probability weight for the occurrence of a collision within the time interval $[t, t+dt]$ with a particle whose state at time t is $2 = (x_2, v_2)$ (see Fig. 1). We can thus write

$$S^R(v_1; t + dt) = S^R(v_1;t)\left\{1 - \int d2\, \theta(x_{21}) \times\theta(x_{12} + v_{12}dt)\mu(2|1;t)\right\}, \tag{7}$$

which implies the equation

$$\frac{\partial}{\partial t}S^R(v_1;t) = -S^R(v_1;t)\int dv_2\, v_{12}\theta(v_{12}) \times\mu(0+, v_2|0, v_1; t). \tag{8}$$

The monotonously decreasing survival probability $S^R(v_1; t)$ can be thus expressed in terms of the density

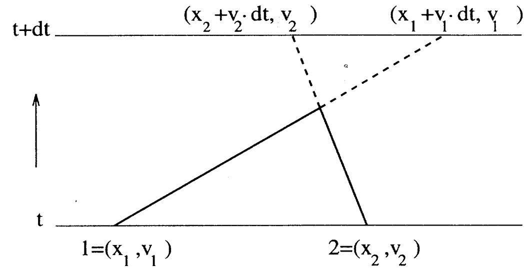

FIG. 1. Annihilating collision between particle 1 and its right nearest neighbor 2 within the time interval $[t, t + dt]$.

$\mu(2|1; t)$ at contact. Introducing the binary collision operator

$$C(1,2) = v_{12}\theta(v_{12})\delta(x_{21} - 0+), \tag{9}$$

where δ is the Dirac distribution, we find that the solution of Eq. (8) satisfying the initial condition $S^R(v_1; 0) = 1$ has the exponential form

$$S^R(v_1;t) = \exp\left\{-\int_0^t d\tau \int d2\, C(1,2)\mu(2|1;\tau)\right\}. \tag{10}$$

The operator $C(1, 2)$ applied to the distribution $\mu(2|1; t)$ determines the collision frequency between particle 1 and its right neighbors 2. In the ballistic case this frequency is proportional to the relative velocity of the colliding pair.

One can analyze the evolution of the survival probability $S^L(v_1; t)$ along the same lines. In the case of a symmetric initial velocity distribution $\phi(v; 0) = \phi(-v; 0)$ (the case considered in this paper) the relation

$$S^L(v_1;t) = S^R(-v_1;t) \tag{11}$$

holds.

III. EVOLUTION OF THE DISTRIBUTION OF NEAREST NEIGHBORS

Equation (10) is an example of a relation which expresses a dynamical property of the fluid in terms of the distribution of nearest neighbors $\mu(2|1; t)$. The relation is important as the density of particles moving with velocity v at time t is given by

$$\sigma(v;t) = \sigma S^L(v;t)S^R(v;t)\phi(v;0),$$

whereas the total density equals

$$\sigma(t) = \int dv \sigma(v;t), \quad \sigma(0) = \sigma. \tag{12}$$

In fact, as will be shown in this section, the evolution of $\mu(2|1; t)$ provides complete information about the process of ballistic annihilation for a large class of initial conditions.

Our object here is to derive the evolution equation determining the distribution $\mu(2|1; t)$ for all $t > 0$. To this end we introduce an infinite set of higher order conditional distributions $\mu_s(2,3,\ldots,s|1;t)$, $s=2,3,\ldots$, with $\mu_2(2|1;t)\equiv\mu(2|1;t)$. The distribution $\mu_s(2,3,\ldots,s|1;t)$ represents the probability density for finding at time t the $(s-1)$ consecutive right nearest neighbors of particle 1 moving with velocities $v_2,v_3,\ldots,v_s$, respectively, at distances

$$0 < x_{21} < x_{31} < \cdots < x_{s1}.$$

The distribution of right neighbors $2,3,\ldots,s$ of particle 1 changes in the course of time as the result of free motion and collisions. The rate of change

$$\frac{\partial}{\partial t}\mu_s(2,3,\ldots,s|1;t)$$

is the sum of contributions yielded by the mechanisms enumerated below. Here is their complete list together with the corresponding analytic expressions.

(i) Free motion:

$$-\left\{\sum_{j=1}^{s} v_j \frac{\partial}{\partial x_j}\right\}\mu_s(2,3,\ldots,s|1;t). \tag{13}$$

(ii) Binary collisions within the ordered sequence $(1,2,\ldots,s)$:

$$-\left\{\sum_{j=2}^{s} C(j-1,j)\right\}\mu_s(2,3,\ldots,s|1;t) \tag{14}$$

[see Eq. (9) and the following comments].

(iii) Destruction of particle s by its right neighbors:

$$-\int d(s+1)\, C(s,s+1)\mu_{s+1}(2,3,\ldots,s,s+1|1;t). \tag{15}$$

(iv) Annihilation of a pair $(s+1,s+2)$ separating particles $(j-1)$ and j:

$$\int d(s+1)\int d(s+2)C(s+1,s+2) \times\sum_{j=2}^{s}\mu_{s+2}(2,\ldots,j-1,s+1,s+2,j,\ldots,s|1;t). \tag{16}$$

(v) Destruction of particle 1 by its right neighbors:

$$\mu_s(2,3,\ldots,s|1;t)\int d(s+1)\, C(1,s+1)\mu(s+1|1;t). \tag{17}$$

The term (17) guarantees the conservation of normalization

$$\int d2\cdots\int ds\, \mu_s(2,3,\ldots,s|1;t) = 1.$$

Indeed, integrating the sum of terms (i)–(iv) over variables $2,3,\ldots,s$ one finds that they induce the decay of the norm of μ_s at the rate

$$-\int d(s+1)\, C(1,s+1)\mu_2(s+1|1;t),$$

which is exactly compensated by the contribution (17).

The dynamics of the ballistic annihilation is thus described by an infinite hierarchy of equations

$$\left\{\frac{\partial}{\partial t}+\sum_{j=1}^{s} v_j\frac{\partial}{\partial x_j}+\sum_{j=2}^{s}C(j-1,j)\right\}\mu_s(2,3,\ldots,s|1;t)$$
$$=\int d(s+1)\,\{\mu_s(2,3,\ldots,s|1;t)C(1,s+1)\mu(s+1|1;t)-C(s,s+1)\mu_{s+1}(2,3,\ldots,s,s+1|1;t)\}$$
$$+\int d(s+1)\int d(s+2)\, C(s+1,s+2)\sum_{j=2}^{s}\mu_{s+2}(2,\ldots,j-1,s+1,s+2,j,\ldots,s|1;t) \tag{18}$$

coupling the conditional distributions μ_s, $s=2,3,\ldots$.

A remarkable property of the hierarchy (18) is its compatibility with the factorization of distributions μ_s for $s>2$ into products of the basic densities $\mu(j|j-1;t)$

$$\mu_s(2,3,\ldots,s|1;t)=\prod_{j=2}^{s}\mu(j|j-1;t),\quad s=3,4,\ldots\ . \tag{19}$$

In order to prove it we insert relations (19) into the hierarchy (18). The first equation ($s=2$) takes then the form

$$\left\{\frac{\partial}{\partial t}+v_1\frac{\partial}{\partial x_1}+v_2\frac{\partial}{\partial x_2}+C(1,2)-\int d3\ [C(1,3)\mu(3|1;t)-C(2,3)\mu(3|2;t)]\right\}\mu(2|1;t)$$
$$-\int d3\int d4 C(3,4)\mu(3|1;t)\mu(4|3;t)\mu(2|4;t)\equiv O_{1,2}\mu(2|1;t)=0, \tag{20}$$

whereas equations corresponding to $s > 2$ can be written [with the use of the nonlinear operator $O_{j-1,j}$ defined in Eq. (20)] as

$$\sum_{j=2}^{s} \prod_{a=2}^{j-1} \mu(a|a-1;t) \, \{O_{j-1,j}\mu(j|j-1;t)\} \times \prod_{b=j+1}^{s} \mu(b|b-1;t) = 0. \tag{21}$$

It follows that the factorized distributions (19) provide a class of solutions of the infinite hierarchy (18) reducing it to the single equation $O_{1,2}\mu(2|1;t) = 0$, provided the relations (19) hold at $t = 0$

$$\mu_s(2,3,\ldots,s|1;0) = \prod_{j=2}^{s} \mu(j|j-1;0). \tag{22}$$

In this way we arrive at the fundamental result of our analysis: the evolution of the fluid from the initial state (22), induced by the ballistic annihilation, preserves the factorized structure of the conditional distributions μ_s, and is entirely described by the kinetic equation (20).

The class of the initial states (22) corresponds to the so-called *renewal processes*: the distributions of the right and left neighbors of a particle are statistically independent. The fact that the annihilation dynamics propagates this property is not surprising. Indeed, the presence of a particle in the fluid excludes any interaction between its right and left neighbors. Notice that condition (22) can be satisfied both by homogeneous and inhomogeneous states. Correlations between the velocities of the nearest neighbors are also allowed.

Let us consider again the initial condition adopted in Sec. II (no correlations between velocities, random distribution in space). The relations (22) then hold with

$$\mu(2|1;0) = \theta(x_{21}) \exp(-\sigma x_{21}) \sigma \phi(v_2). \tag{23}$$

The exponential factor $\exp(-\sigma x_{21})$ is the Poisson probability for finding an empty interval of length $x_{21} > 0$ when the number density of the fluid is σ.

Using the explicit form (9) of the binary collision operator $C(1,2)$ and the translational invariance of the fluid we rewrite Eq. (20) in the region $x = x_{21} > 0$ as

$$\left\{\frac{\partial}{\partial t} + v_{21}\frac{\partial}{\partial x}\right\} \mu(x,v_2|0,v_1;t)$$
$$= \mu(x,v_2|0,v_1;t) \int dv_3 \, [v_{13}\theta(v_{13})\mu(0+,v_3|0,v_1;t) - v_{23}\theta(v_{23})\mu(0+,v_3|0,v_2;t)]$$
$$+ \int dv_3 \int dv_4 \int_0^x dy \mu(y,v_3|0,v_1;t)\mu(x-y,v_2|0,v_4;t) v_{34}\theta(v_{34})\mu(0+,v_4|0,v_3;t). \tag{24}$$

The solution of Eq. (24) determines the distribution $\mu(x,v_2|0,v_1;t)$ for distances $x > 0$. The product $\theta(x)\mu(x,v_2|0,v_1)$ yields then the solution of Eq. (20) in which the possibility of $x = 0+$ is taken into account [term $C(1,2)\mu(2|1;t)$].

The convolution structure of the last term in Eq. (24) suggests the application of the Laplace transformation. Putting

$$\tilde{\mu}(z,v_2|v_1;t) = \int_{0+}^{\infty} dx \, \exp(-xz)\mu(x,v_2|0,v_1;t) \tag{25}$$

we find

$$\left\{\frac{\partial}{\partial t} + zv_{21} + \frac{\dot{S}^R(v_1;t)}{S^R(v_1;t)} - \frac{\dot{S}^R(v_2;t)}{S^R(v_2;t)}\right\} \tilde{\mu}(z,v_2|v_1;t) - v_{21}\mu(0+,v_2|0,v_1;t)$$
$$= \int dv_3 \int dv_4 \, \tilde{\mu}(z,v_3|v_1;t)\tilde{\mu}(z,v_2|v_4;t) v_{34}\theta(v_{34})\mu(0+,v_4|0,v_3;t). \tag{26}$$

In writing Eq. (26) we used the relation (8) and the shorthand notation

$$\dot{S}^R(v;t) = \frac{\partial}{\partial t} S^R(v;t).$$

Equation (26) is a convenient starting point for the analysis of the process of ballistic annihilation in translationally invariant states.

IV. THE TWO-VELOCITY CASE

In this section we solve the basic kinetic equation (26) in the case where the particles can propagate with only two possible velocities $+c$ or $-c$, and the initial velocity distribution is symmetric. Using the Kronecker delta function δ^{Kr} rather then the Dirac distribution we thus put

$$\phi(v;0) = \frac{1}{2}[\delta^{\rm Kr}_{v,c} + \delta^{\rm Kr}_{v,-c}]. \tag{27}$$

The symmetry relation (11) is satisfied, which permits us to introduce a simplified notation

$$S(t) = S^R(+c;t) = S^L(-c;t).$$

Clearly, $S^R(-c;t) = S^L(+c;t) = 1$, as no particle moving with velocity $-c$ $(+c)$ can be attained by its right (left) neighbors. So $S(t)$ is the complete survival probability of a free trajectory [compare with formula (4)].

Consider first the equation satisfied by $\tilde{\mu}(z,+c|-c;t)$. From Eq. (26) we find

$$\left\{S(t)\left[\frac{\partial}{\partial t} + 2cz\right] - \dot{S}(t)\right\}\tilde{\mu}(z,+c|-c;t)$$
$$= -\dot{S}(t)[\tilde{\mu}(z,+c|-c;t)]^2. \tag{28}$$

Equation (28) implies a linear equation for $[\tilde{\mu}(z,+c|-c;t)]^{-1}$, and can be thus readily solved. One finds

$$\tilde{\mu}(z,+c|-c;t) = \frac{\sigma}{2}S(t)\exp(-2ctz)\tilde{A}(z;t), \tag{29}$$

where

$$\tilde{A}(z;t) = \left[z + \sigma + \frac{\sigma}{2}\int_0^t d\tau \exp(-2c\tau z)\dot{S}(\tau)\right]^{-1}. \tag{30}$$

In writing the solution (29) the initial condition [see Eq. (23)]

$$\tilde{\mu}(z,+c|-c;0) = \sigma/2(z+\sigma)$$

has been used.

One can get a clear physical interpretation of the inverse Laplace transform $A(x;t)$ of function (30) by considering the equation satisfied by $\tilde{\mu}(z,+c|+c;t)$. Indeed, from Eq. (26) we get

$$S(t)\frac{\partial}{\partial t}\tilde{\mu}(z,+c|+c;t)$$
$$= -\dot{S}(t)\tilde{\mu}(z,+c|-c;t)\tilde{\mu}(z,+c|+c;t). \tag{31}$$

Inserting here the solution (29) one finds a simple formula

$$\tilde{\mu}(z,+c|+c;t) = \frac{\sigma}{2}\tilde{A}(z;t). \tag{32}$$

Now, the kinetic equation (26) shows that the distribution $\tilde{\mu}(z,+c|+c;t)$ changes in the course of time through one and only one mechanism: mutual annihilation of particles originally separating the pair $(+c,+c)$ [only the term on the right hand side of (26) contributes]. It follows that the inverse Laplace transform

$$A(x;t) = \int \frac{dz}{2\pi i}\exp(xz)\tilde{A}(z;t) \tag{33}$$

represents the probability that all particles present initially in an interval of length x disappear through ballistic annihilation before time t [the prefactor $\sigma/2$ in Eq. (32) is the initial density of the right neighbors moving with velocity $+c$].

We know that the colliding pairs move with a relative velocity $2c$. So, all sequences of encounters contributing to $A(x;t)$ are accomplished at the moment $t^* = x/2c$. Therefore $A(x;t)$ does not depend on time (for fixed x) in the region $t > t^*$. This important conclusion applies also to $\mu(x,+c|0,+c;t)$ [see Eq. (32)].

For symmetry reasons $\tilde{\mu}(z,+c|+c;t) = \tilde{\mu}(z,-c|-c;t)$. It remains thus to determine the function $\tilde{\mu}(z,-c|+c;t)$ which contains all the information about the collision rate in the fluid. From the basic Eq. (26) one gets

$$\left\{S(t)\left[\frac{\partial}{\partial t} - 2cz\right] + \dot{S}(t)\right\}\tilde{\mu}(z,-c|+c;t)$$
$$= +\dot{S}(t)[\tilde{\mu}(z,+c|+c;t)]^2. \tag{34}$$

Inserting here the solution (32) after a straightforward calculation we find

$$\tilde{\mu}(z,-c|+c;t) = \left\{\frac{\sigma}{2}\exp(2ctz)\tilde{A}(z;t) + \int_0^t d\tau \exp[2cz(t-\tau)]\dot{S}(\tau)\right\}[S(t)]^{-1}. \tag{35}$$

In order to find the time dependence of the survival probability $S(t)$ we use the inverse Laplace transform at $x = 0^+$

$$\mu(0+,-c|0,+c;t) = \lim_{0<x\to 0}\int \frac{dz}{2\pi i}\exp(xz)\tilde{\mu}(z,-c|+c;t). \tag{36}$$

In view of Eq. (8), which takes here the form

$$\dot{S}(t) = -2cS(t)\mu(0+,-c|0,+c;t),$$

Eq. (36) leads to a closed consistency equation for $S(t)$

$$\dot{S}(t) = -2c\int \frac{dz}{2\pi i}\left\{\exp(2ctz)\frac{\sigma}{2}\tilde{A}(z;t) + \int_0^t d\tau \exp[2cz(t-\tau)]\dot{S}(\tau)\right\}. \tag{37}$$

In the first term in the right hand side of Eq. (37) the inverse Laplace transformation yields the value of function $A(x;t)$ at the point $x = 2ct$. From our previous analysis following Eq. (33) we know that by replacing $\tilde{A}(z;t)$ in Eq. (37) by $\tilde{A}(z;\infty)$ we do not change the value of the integral. From formula (30) we get

$$\tilde{A}(z;\infty) = \left[z + \sigma + \frac{\sigma}{2}[2cz\tilde{S}(2cz) - 1]\right]^{-1},$$

where $\tilde{S}(z)$ is the Laplace transform of $S(t)$. As the contribution from the last term in the right hand side of Eq. (37) vanishes we eventually arrive at the simple relation

$$z\tilde{S}(z) - 1 = -\left[\frac{z}{c\sigma} + z\tilde{S}(z) + 1\right]^{-1}. \tag{38}$$

The physically relevant solution of Eq. (38) reads

$$z\tilde{S}(z) = -\frac{z}{2c\sigma} + \sqrt{\left(\frac{z}{2c\sigma}+1\right)^2 - 1}.$$

Inverting the Laplace transformation one recovers the Elskens and Frisch result [3]

$$S(t) = \exp(-2ct\sigma)[I_0(2ct\sigma) + I_1(2ct\sigma)], \tag{39}$$

where I_0 and I_1 are the modified Bessel functions. The long time decay of the number density $\sigma(t)$ [see Eq. (12)] is governed by the power law

$$\sigma(t) = \sigma S(t) \simeq \sqrt{\frac{\sigma}{\pi ct}}, \quad t \to \infty.$$

Equations (29), (30), (32), and (37), combined with the formula (39), determine entirely the distribution of the right nearest neighbors in the fluid at any time $t > 0$. As the higher order distributions remain factorized, the complete dynamical description of the system is in this way achieved [the distribution $\mu(2|1;t)$ is discussed in [3] only for $t = \infty$].

As an example of an interesting question concerning the annihilation dynamics let us evaluate the mean velocity of the right nearest neighbor of a particle moving with velocity $+c$. At $t = 0$ this conditional mean velocity equals zero. For $t > 0$, from Eqs. (32) and (35) we obtain

$$c\tilde{\mu}(0,+c|+c;t) - c\tilde{\mu}(0,-c|+c;t) = c\left[\frac{1-S(t)}{1+S(t)}\right], \tag{40}$$

where the relation $\sigma\tilde{A}(0;t) = 2[S(t)+1]^{-1}$ has been used [see Eq. (30)]. The above formula shows that in the long time limit it is much more likely to find the right nearest neighbor moving in the same direction (with velocity $+c$) than in the opposite one. This intuitively clear prediction reflects the building up of correlations between the velocities in the process of ballistic annihilation.

V. CONCLUDING REMARKS

The analytic approach developed here culminated in the derivation of the kinetic equation (24) for the two-particle distribution of the right nearest neighbors $\mu(2|1;t)$. The annihilation dynamics turns out to be incompatible with the Boltzmann molecular chaos assumption as the correlations between the velocities of colliding pairs are of primary importance. This can be clearly seen in the example of the two-velocity case solved in Sec. IV. Although the one-particle velocity distribution does not change in the course of time (by symmetry velocities $+c$ and $-c$ have always equal probabilities), the conditional distribution $\tilde{\mu}(0,v_2|v_1;t)$ does change, favoring configurations in which the nearest neighbors move in the same direction: $v_2 = v_1$ [see the discussion following Eq. (40)]. It is thus even more remarkable that for a large class of initial conditions satisfying relations (22) the infinite dynamical hierarchy (18) propagates the factorization (19) of higher order distributions and reduces to a single kinetic equation (24) for $\mu(2|1;t)$.

We have shown in Sec. IV how to recover the results of the combinatorial approach in the two-velocity case by solving the kinetic equation. However, the derived equation (24) can be applied to any initial velocity distribution $\phi(v;0)$, and presents thus a promising basis for the analysis of ballistic annihilation in the case of more realistic velocity spectra.

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